Interdisciplinary Aspects of the Teaching and Learning of Mathematical Modelling in Mathematics Education Including Relations to the Real World and STEM[1]

Gloria Ann Stillman[2], Toshikazu Ikeda[3], Stanislaw Schukajlow[4], Jussara de Loiola Araújo[5], and Jonas B. Ärlebäck[6]

ABSTRACT This report presents preliminary results from a survey commissioned for ICME-14 on *teaching and learning of mathematical modelling and interdisciplinary mathematics educations.* The systematic literature review focusses on how a well-understood relation between mathematics and the real world underpins interdisciplinary work, interdisciplinarity in research and teaching teams, issues and challenges in the relationships among mathematical modelling, mathematics, the real world and interdisciplinarity, and mathematical modelling and a well-understood relation to the real world ensuring mathematical depth in STEM integration.

Keywords: Interdisciplinary; Mathematics; Mathematical modelling; Relations to the real world; STEM.

Preprint

1. Background

1.1. The survey team's terms of reference

This paper reports preliminary findings from Survey Team 4, commissioned for ICME-14, focusing on the topic, *The teaching and learning of mathematical modelling and interdisciplinary mathematics educations,* expanding on aspects presented by the authors during the congress in July 2021. The terms of reference of Survey Team 4 were the teaching and learning of mathematical modelling and interdisciplinary mathematics education with a focus on relations to the real world and connections and

[1] The title appeared in the Program Brochure is *On the Teaching and Learning of Mathematical Modelling and Interdisciplinary Mathematics Education.*

[2] School of Education (Victoria), Australian Catholic University (Ballarat Campus), Ballarat, Victoria 3350, Australia. E-mail: gloria.stillman@acu.edu.au

[3] College of Education, Yokohama National University, Yokohama, 240-8501, Japan. E-mail: toshi@ynu.ac.jp

[4] Department of Mathematics and Computer Science, University of Münster, D-48149 Münster, Germany. E-mail: schukajlow@uni-muenster.de

[5] Department of Mathematics, Federal University of Minas Gerais, MG, 31270-901, Brazil. E-mail: jussara@mat.ufmg.br.

[6] Department of Mathematics, Linköping University, Linköping, Sweden. E-mail: jonas.bergman.arleback@liu.se

implications for Science, Technology, Engineering and Mathematics (STEM) education.

The importance of mathematical modelling and mathematical applications to real world context has been growing in mathematics education over recent decades supported by regular activities on applications and modelling at the ICME's and through the series of International Conferences on the Teaching of Mathematical Modelling and Applications (ICTMAs), held biennially since 1983 (except for the recent interruption due to the COVID-19 pandemic). ICME Proceedings and Survey Lectures indicate the most recent developments at the relevant time and contain many studies, conceptual contributions, and resources addressing the relation between the real world and mathematics, as do the ICTMA Series books (e.g., Leung et al., 2021). Additionally, ICMI study 14 on *Modelling and Applications in Mathematics Education* (Blum et al., 2007) addressed a variety of topics related to modelling bringing together international perspectives. An increasing number of empirical research projects which focus on special aspects of mathematical modelling and applications, as well as national and international comparative studies, generate further interest.

In 1989, D'Ambrosio proposed that mathematical modelling is the thread by which individual disciplines in the curriculum can be interconnected to promote curricular integration. English has long argued (e.g., English 2007, 2008; English et al., 2016) for mathematical modelling becoming an enabler of interdisciplinary practices in schools, as the means by which "creative and flexible use of mathematical ideas within an interdisciplinary context where students solve substantive, authentic problems that address multiple core learnings" is promoted (2007, p. 275). The survey therefore reviews the current state-of-the-art on the teaching and learning of mathematical modelling under the specific consideration of interdisciplinary aspects. For this reason, a well understood relation between mathematics and the real world is an important focus. This is particularly relevant in the context of STEM, which has recently come to political prominence (Kelley and Knowles, 2016; Moore et al., 2020) in several educational jurisdictions around the world.

1.2. Interdisciplinary approaches to education and research

The construct, *interdisciplinarity*, entered the literature in 1972 (Miller, 2020) in an Organization for Economic Cooperation and Development report (Apostel, 1972), although the idea was well known in education long before this. Like most constructs, interdisciplinarity has morphed to have several meanings. In our survey, we were guided by the USA National Academies definition for *interdisciplinary research*:

> a mode of research by teams or individuals that integrates information, data, techniques, tools, perspectives, concepts, and/or theories from two or more disciplines or bodies of specialized knowledge to advance fundamental understanding or to solve problems whose solutions are beyond the scope of a single discipline or area of research practice. (National Academy of Sciences, 2005, p. 39)

Preprint

In actual teaching and learning situations in classrooms, different approaches to integration can be seen as *isolated* into separate disciplines, or *connected* deliberately relating separate disciplines to show connections, or *nested/fused* where content from one or more other disciplines is taught in a discipline to enrich it, or *multidisciplinary* incorporating two or more disciplines around the same theme or topic but the disciplines keep their identity, or *interdisciplinary* when two or more disciplines interact to become something new such as mathematical ecology, or *transdisciplinary* where there is a transcendence of the disciplines and the focus becomes the field of knowledge as exemplified in the real world (Gresnigt et al., 2014, p. 52). Gresnigt et al. use the metaphor of a staircase of increasing complexity where more elements of teaching are shared between disciplines rather than assuming the higher up the ladder the better the integration. Instead, they characterise these steps as different ways to integrate. Others such as Williams and Roth (2014) include *meta-disciplinarity* where there is awareness of the nature of the disciplines involved in their relation and the differences within an inquiry or problem solving. As Roehrig et al. (2021) point out, "It is the multidisciplinary nature of real-world problems, as opposed to the disciplinary structure within formal schooling, that grounds arguments for curricular integration" (p. 2).

1.3. Mathematical modelling and interdisciplinarity

Mathematical modelling can be understood as real-world problem solving, although we acknowledge it does not have to be. Mathematical modelling in this interpretation is then the process of applying mathematics to a real-world problem or situation with the goal of understanding it (Niss et al., 2007). It is more than applying mathematics in a closed situation where nothing needs to be assumed or estimated — just a known mathematical technique applied. Thus, multiple interpretations of the situation being modelled are possible. "The modelling enterprise involves identifying and addressing open-ended questions, creating, refining and validating models, and arguing the case for implementation of model informed outcomes" (Niss et al., 2007, p. 17).

In addition, from an epistemological perspective, "essential characterisations of modelling… involve posing and solving problems located in the real-world, which for our purposes includes other discipline areas such as engineering or medicine, and general contexts of living as they impact on individuals, groups, and communities" (Niss et al. 2007, p. 17). Researchers have therefore developed or proposed theoretical views on the interplay between mathematical modelling and interdisciplinarity in mathematics education (e.g., Borromeo Ferri & Mousoulides, 2017; English, 2013; Michelsen, 2006, 2015).

English (2013), for example, argues that there is a need to build a stronger foundation in the mathematical sciences through future-oriented learning experiences to equip students for the challenges of the 21st century. She lists core competencies that are key elements of productive and innovative workplace practices to ensure such a foundation. To achieve this aim, she recommends an increased focus on interdisciplinary problem solving that engages students in complex modelling with challenging, life-based scenarios. In such learning experiences, knowledge and skills

Preprint

from at least two disciplines are applied to real-world scenarios with the aim of shaping the total learning experience. English sees such mathematical modelling having applicability in primary and middle schooling where the focus is on engaging students in the kinds of mathematical and scientific thinking needed for challenges beyond the classroom. She supports her argument from two design-based studies, a data modelling study in Year 1 and engineering-based modelling experiences in Year 7.

1.4. Mathematical modelling and a well understood relation to the real-world

Relations between mathematics and the real world have existed since the very beginnings of mathematics (see, e.g., Joseph, 2011). If mathematical modelling is conceived as real-world problem solving, a well understood relation between mathematics and the real world is essential. When such a relationship is in play, the real-world situation encourages a deeper understanding and processing of mathematics; but simultaneously use of mathematics encourages deeper understanding and processing of the real world. Each enriches the other.

The example of water falling from two gates on a dam wall into the spillway below illustrates our point and is a context to design modelling materials for a secondary school classroom. From a mathematical point of view, the maintaining of a well-understood relation of any in-class modelling to the real world is an issue for teaching mathematics in school. The situation to be modelled is derived from the observation that the water from the gate that is lower in the dam wall appears to have stronger momentum than that from the higher gate; however, the horizontal distances from the dam wall to the impact point on the spillway seem to be similar. We pose the real-world problem: What is the relation between the location of a gate and the horizontal distance of where the water lands? This situation is concerned with mathematics and physics. (See Stillman et al., submitted, for a full solution to this problem.)

From a mathematics perspective, students realise the importance of generating and selecting variables, setting up a simplified situation and validating the solution derived from the model by experiment. During their modelling, students have opportunity to appreciate the utility of mathematics to understand (represent, explain, predict) parts of the world. From a real-world perspective, knowledge of physics is enriched. Students learn that the velocity of the spilling water is proportional to the square root of the distance of the gate from the top and that the horizontal distance of the landing site of the spilling water becomes a maximum when the gate is located at the mid-point of the height (depth) of the water in the dam at the wall where the gates are located.

Two questions are crucial in both solving an authentic real-world task like this and in planning its implementation and management in class. Firstly, what type of mathematics can be applied and secondly, how can the real-world situation be conceptualised. Interaction between these two leads to a well-understood relation between mathematics and the real world *when they enrich each other*.

2. Focuses and Research Questions

It is timely for the international mathematics education community to survey, synthesise, and propose new directions for research that is focused on interdisciplinary

Preprint

aspects of research and teaching in mathematical modelling. The survey team has been addressing the specific scope and foci of relevant work that has been developing in these areas over the last decade in different educational systems around the world. The interdisciplinarity aspects that have been focused on are (a) interdisciplinarity in research teams as well as research focussed on interdisciplinarity and (b) interdisciplinarity in teaching and teaching design teams through all levels of schooling and into tertiary education.

Broad research questions, guiding the analysis the survey team has conducted for this paper are:

- How does a well-understood relation between mathematics and the real world underpin interdisciplinary work in mathematics education?
- How have interdisciplinary teams contributed to knowledge about mathematical modelling and the relation of mathematics to the real world?
- What issues and challenges are there in the relationships among mathematical modelling, mathematics, the real world and inter-disciplinarity in both teaching and research?
- How could contributions from research and teaching on mathematical modelling and relations of mathematics to the real world contribute to ensuring mathematical depth in STEM integration?

3. Methodology

The team started collection and collation of potential sources for the survey by calling for contributions on several online list serves which resulted in contributions by individuals of lists of publications, including projected future publications, and chapters about research and teaching projects with interdisciplinary connections of mathematics as well as mathematical modelling. An initial surveying of literature (including English, German, Japanese, Portuguese and Swedish) from selected geographical regions was conducted by different team members. This established a basis for what we might expect to locate in the time period and the likely fruitfulness of potential sources, so a systematic survey of sources began in preparation for a systematic analytical review of literature (Newman and Gough, 2020). Synopses of all selected sources were collated in one database where they were coded by reading the original source and the synopsis, re-reading the source and adding to the synopsis when necessary. Sources that did not appear to be within the remit of the terms of reference were starred for potential culling following cross checking. This was carried out by two team members independently. Our initial focus was on the period 2012–2020 but then we conducted a more in-depth search of 2016–2021, once the date of the conference and reporting was extended. Our sources included refereed journals, edited books (especially in relevant book series in mathematics education and other fields such as STEM), conference proceedings, and theses (see Tab. 1). Our database extends beyond this because of the initial cross geographical regions search and individual contributions. As well as major mathematics education journals, we have read and collated articles from other journals to augment understanding of the breadth and depth of research and teaching in other fields such as science education and to gain insight

Preprint

into the STEM field, especially in relation to mathematical modelling outside of what is usually cited in mathematics education. We, thus, examined articles from the *International Journal of STEM Education*, amongst other journals.

Tab. 1. Literature sources systematically surveyed

Sources
Journals
Educational Studies in Mathematics 2016–2021
Eurasia Journal of Mathematics Science and Technology Education 2018–2020
European Journal of Science and Mathematics Education 2016–2021
Journal for Research in Mathematics Education 2016–2021
Journal of Mathematical Behavior 2016–2021
Journal of Science Education in Japan
International Journal of Science and Mathematics Education 2016–2021
International Journal of STEM Education 2016–2021
Mathematical Thinking and Learning 2016–2021
Mathematics Education Research Journal 2016–2021
Nordic Studies in Mathematics Education
ZDM — Mathematics Education 2016–2021
Book Series in Mathematics Education
NCTM Monographs APME 1 volume (Hirsh and Roth McDuffie, 2016)
Realitätsbezüge im Mathematikunterricht book series (REIMA) 11 volumes (e.g., Frank et al. in Greefrath & Siller, 2018)
Springer Series: *Advances in Mathematics Education* — 1 volume (Chamberlin and Sriraman, 2019)
Springer Series: *Early Mathematics Learning and Development* — 1 volume (Suh, Wickstrom and English, 2021)
Springer Series: *ICME-13 Monographs* — 2 volumes
(Doig, Williams, Swanson, Borromeo Ferri and Drake, 2019; Stillman and Brown, 2019)
Springer Series: *International Perspectives on the Teaching and Learning of Mathematical Modelling* — 4 volumes (Stillman, Kaiser, Blum and Brown, 2013; Stillman, Blum and Biembengut, 2015; Stillman, Blum and Kaiser, 2017; Stillman, Kaiser and Lampen, 2020; Leung, Stillman, Kaiser, and Wong, 2021)
Books from other fields — examples
Comparison of mathematics and physics education I: Theoretical foundations for interdisciplinary collaboration (Kraus and Krause, 2020)
Asia-Pacific STEM teaching practices (Hsu and Yeh, 2019)
Handbook of research on STEM education (Johnson, Mohr-Schroeder, Moore and English, 2020)
Conference and Symposium Proceedings
CERME 8, CERME 9, CERME 10, CERME 11
CNMEM 8^{th}, 9^{th}, and 10^{th} editions of the National Conference on Modelling in Mathematics Education (Conferência Nacional sobre Modelagem na Educação Matemática) — Brazil 28 works analysed
MACAS 2017; MERGA 2016–2019; NORMA17; PME 43, 42, 41, 40; PME-NA 41
Research projects
Funded research 2011–2021 projects in mathematics education in the Nordic countries also involving interdisciplinarity and STEM
Theses and dissertations
Craig (2017); Gibbs (2019)

In the grey literature we examined outputs in recent years in several conference series in different parts of the world and added those where there was no similar

Preprint

literature in other scholarly sources. Research projects, both by established researchers and teams and by early career researchers in theses and dissertations, were also surveyed. For example, when Prof. Arleback joined our team at the end of 2020, he began by surveying funded research projects in mathematics education in the Nordic countries which also involved interdisciplinarity and STEM to gain an understanding of what was happening in the time period of the survey. It was expected this would lead to published literature on the earlier projects but most likely not on the recent ones, given COVID-19 had restricted conference travel and possibly the research proceeding.

A coding scheme was developed, and the codes for this paper were structured around our research questions according to four overarching categories: (a) relations between mathematics and the real world (RQ 1), (b) interdisciplinary team contributions (RQ2), (c) issues and challenges in relationships (RQ3), and (d) mathematical depth in STEM integration (RQ4). Tab. 2 provides an example of our coding scheme for a selection of the reviewed literature for the interdisciplinary team contributions category. Similar coding schemes were developed for the other three categories. Initial and final coding was carried out by the first author. Another member of the team checked the first coding and sent all queries to the initial coder who then carried out a second coding. Subsequently a third re-coding round was conducted 6 months later to check coding reliability. A configurative synthesis (Newman and Gough, 2020) of the different literature sources was then conducted to answer our research questions, focusing on the research questions and problems or topics the selected literature addressed, noting confirmatory and contradictory findings and rival explanations of such findings.

Tab. 2. Example coding scheme for interdisciplinary team contributions category

Code	Brief Description	Examples of Literature
CoA	Co-authorship from other domain(s) other than mathematics education	Sala et al. (2017); Sawatzki et al. (2019); Viirman and Nardi (2021)
CoA MN	Co-authorship multi-national team	Chang et al. (2020); Frejd and Geiger (2017); Guerrero-Ortiz et al. (2016)
Contribution – CC	Cross cultural validation	Durandt et al. (2022)
Contribution – IC	International comparison	Chang et al. (2020)
Contribution – KT	Knowledge transfer between countries	Krawitz et al. (2022)
IDT	Interdisciplinary design team	Viirman and Nardi (2021)
IRT	Interdisciplinary research team	Durandt et al. (2022)
ITT	Interdisciplinary teaching team	Gardner and Tillotson (2019)
MM	Contribution to mathematical modelling research/ teaching/ design/curriculum	Gardner and Tillotson (2019); Viirman and Nardi (2021)
R to RW	Consideration of/new contribution to research on/ relations to real world	Guerrero-Ortiz et al. (2016); Viirman and Nardi (2021)

4. Findings

We present our findings in four threads to address our four research questions where overall trends, issues and challenges will be illustrated and exemplified.

Preprint

4.1. *Relation between mathematics and the real world underpinning interdisciplinary work in mathematics education*

As seen in subsection 1.4, to try to understand the relation between mathematics and the real world, we think about two worlds, the real world and mathematics. On the one hand, the real world encourages deeper understanding and processing of mathematics. On the other hand, mathematics encourages deeper understanding and processing of the real-world situation. If both are satisfied, enriching each other, we say the relation between mathematics and the real world is well-understood and this is what we see as ideal to underpin interdisciplinary work in mathematics education.

Mathematical modelling is sometimes described as a transformation of a real-world problem to a mathematical problem and back again, so the real-world and mathematical world appear distinct. Czocher (2018) raises and considers a critical issue in the modelling process related to a deep understanding of the *relation of mathematics to the real world*: “How do modellers determine if the transformation from the real world to mathematics was conducted well?”

From a study of the modelling activity of four engineering students, Czocher (2018) presents an empirically derived typology of five validating activities to explain how validating functions to ensure a mathematical model will yield a reasonably accurate prediction. To capture the complexity of modelling due to validating, she offers an empirically grounded schematic adapted from the modelling cycle diagram of Blum and Leiß (2007). Two circular regions show the real world and the mathematical world, respectively; however, a honeycomb-patterned annulus surrounds the mathematical world representing reasoning that is mathematically structured but constrained by real-world conditions. Triangles and circles are used to indicate the model construction stages (e.g., situation model) with solid arrowed arcs showing the transitions between these. Dotted arrowed arcs show validating actions which lead to, or from, the mathematically structured real world. The schematic shows that most of the observable reasoning occurs in the annulus as a blend of the real-world and the mathematical world, thus validating ensures the real world and mathematical world stay intertwined. Czocher (2018) proffers this model because she found no evidence of a separation into purely mathematical thinking and purely real-world thinking and switching between the two in her study.

The four validating activities that perform this function are comparing the mathematical expression, its constituent components or relationships, to the interpretation of the problem setting; comparing the mathematical expression, its constituent components or relationships, to the idealized version of the problem setting; comparing the real results to empirical, or based-on-empirical, expectations (predicted by theory); and comparing real results against physical principles accounted-for in the real model. The fifth validating action she identified occurred purely in the mathematical world. “The nuances of validating suggest that creating and maintaining relationships between reality and mathematics [are] more complex than a transformation” (p. 137). As a consequence, Czocher (2018) suggests a more prominent role should be afforded to validation in the modelling process. Czocher et al. (2018) argue that in order to foster learners who are confident and capable in STEM

Preprint

fields, it is necessary to revisit how verifying and validating activities are conceptualised and developed across the years of schooling and in different subject areas. (See also Jensen, 2018.)

Another issue in teaching modelling is how to facilitate students' conceptualisation of the real-world situation. Bearing this in mind, Wernet (2017) focused on the interaction between teachers and students about contextual features in written tasks. During whole class discussions, teachers and students discussed context of written problems in multiple ways, and these interactions often led to higher authenticity (Palm 2008) being enacted in discussions, than was written in the task descriptions. In a similar vein, Chang et al. (2020) focused on making assumptions to conceptualise a real-world situation. Such conceptualisation necessarily involves both realistic considerations and non-mathematical knowledge. Inadequate assumption making leads to an inadequate situation model and to an inadequate mathematical model for the problem situation.

4.2. Contributions from interdisciplinary teams to knowledge about teaching and learning mathematical modelling and relations to real-world

With respect to contributions from interdisciplinary teams, there were some limitations with respect to determining disciplines of researchers and team composition from published work and how, and to what extent, a researcher contributed to a study. At times this information was available in the biographies of authors or in acknowledgements but at other times information had to be sought from project and university staff personal webpages. In many cases, many experts who contributed to a project are not visible (e.g., curriculum designers, methodology experts). Despite these limitations, there was evidence of contributions of several interdisciplinary research teams and interdisciplinary teaching and curriculum design teams and combinations of these to knowledge about teaching and learning of mathematical modelling and relations to the real-world. Some of the interdisciplinary teams were multi-national, whereas others addressed topics in which interdisciplinary teams are essential. The latter included international comparative studies where both culture and language had to be taken into account, knowledge transfer from one country to others, and validation of results of a study across countries and cultures.

The study by Chang et al. (2020), for example, involved a team of four mathematics educators, two from Taiwan and two from Germany. The aim was to compare Year 8 Taiwanese and Year 9 German secondary school students' knowledge use in solving structured modelling problems set in familiar contexts. In this study, the researchers intentionally designed two types of assumptions in two modelling tasks, namely, one requiring only non-numerical assumptions and another requiring both non-numerical and numerical assumptions. As few current studies comparing modelling performance between Western and non-Western students have considered the differences in students' knowledge, students' relative performance was investigated when students' mathematical knowledge in solving modelling problems was matched. The results showed that the Taiwanese students had significantly higher

Preprint

mathematical knowledge than did the German students, whether conceptual or procedural. However, when students had the same level of mathematical knowledge, the German students showed higher modelling performance on the same modelling problems, no matter what type of assumptions were necessary. Chang et al. suggest that their findings imply that Western mathematics education may be more effective in improving students' ability to solve holistic modelling problems. It should be noted, though, that the German school mathematics curriculum provided opportunities for students to learn modelling, whereas the Taiwanese one did not, so the findings could also be reflecting opportunity to learn.

Viirman and Nardi (2021) report on the work of an interdisciplinary research team comprised of three tertiary mathematics education researchers and one research mathematician with extensive experience of mathematical modelling and teaching applied mathematics. Mathematical modelling was being used as a vehicle to integrate mathematics and biology to improve tertiary biology students' engagement with mathematics and their competencies in both mathematics and biology. Viirman and Nardi traced the students' meta-level learning about mathematical modelling, particularly as they fluctuated between deploying graphs for mere illustration of data and as sense-making tools. Some students, however, used graphing only for illustration of work done and not to make meaning from data in their modelling. Previous mathematical experiences with graphing were relied on and the necessity to maintain a good relation, between mathematics and the real or extra-mathematical worlds in the situations they were modelling, was not noticed. Hankeln (2020) noted similar results from a comparative study of upper secondary school students' modelling processes in Germany and France. Although French students were more unfamiliar than German students with real-world context being of more importance than as a mere motivation to engage in mathematics, German students often just exposed the mathematical content as they had accepted socio-mathematical norms that a deep understanding of the real-world context was not the focus of mathematical tasks and could even be a hinderance. These findings in both studies highlight the importance of teachers at the secondary and tertiary levels making students aware of the depth of engagement with real-world aspects of tasks that are needed to be productive in adjusting modelling methods and models used previously to fit a new situation when constructing, interpreting, and validating models or the modelling used.

4.3. Issues and challenges in the relationships among mathematical modelling, mathematics, the real world and interdisciplinarity

Several issues and challenges in the relationships among mathematical modelling, mathematics, the real world and interdisciplinarity in teaching and research were raised in the literature reviewed. We have space to raise only a few.

As Carreira and Baioa (2015) point out, it is rare in mathematics classes for secondary students to engage in examining material objects and artefacts of the real world to generate mathematical ways of viewing reality. However, such experiences enable the making of connections between students' everyday life, mathematics, and the real world. Carreira and Baioa designed and implemented a mathematical

Preprint

modelling learning activity where students collected data about staircases in their neighbourhood. This allowed students to come to the realisation that linearization was the key to designing the best staircase for a house despite finding slight variations from the ideal in reality. A linear model came from mathematical notions such as average step to compensate for variability in tread and riser measurements in producing a staircase of constant slope. As Diego-Mantecón, Haro et al. (2021) note, in teaching activities such as this, there is a need for contextualised knowledge that is not taught in school but, as Carreira and Baioa (2015) have demonstrated, there are ways to gain some of this such as going out into the locality where students live.

In other work, Diego-Mantecón, Prodromou et al. (2021) found that in-field mathematics teachers avoided using transdisciplinary projects where school mathematics content was difficult to address, whereas out-of-field mathematics teachers tended to overlook the mathematics in interdisciplinary projects, oversimplified it, or used basic mathematics below curriculum standards. In-field mathematics teachers promoted high cognitive demand and productive dispositions towards mathematics in projects. Some students, however, did not want to invest time into such projects, being more examination oriented. Students also contributed to projects where their strengths were (e.g., practical skills) rather than use the opportunity to improve areas where they were less confident (e.g., mathematical analysis).

4.4. Ensuring mathematical depth in STEM integration

STEM integration can foreground mathematics teaching, learning and use in two ways: *intra-mathematical* providing opportunities to engage with mathematics in the development and application of mathematical ideas, concepts and skills in the context of meeting curriculum obligations in mathematics or *extra-mathematical* as a unique mathematical viewpoint and its practice contributing to understanding and development of ideas, concepts and skills in other STEM disciplines. According to several authors, the optimisation of these ways of foregrounding mathematics so there is a noticeable effect on mathematical achievement and learning is yet to be realised as mathematics is seen as benefiting least from STEM integration (English, 2016; Fitzallen, 2015; Maass et al., 2019). A study by Li et al. (2020) of publicly funded STEM projects in the USA from 2003-2019 perplexingly showed that the majority of projects focused on single disciplines, especially mathematics. However, these projects showed a strong emphasis on mathematics, particularly before 2012.

To address the rather shallow treatment of mathematics in STEM education, several sources (e.g., Ärlebäck and Albarracin, 2019; English et al., 2016; Gonçalves and Pires, 2014; Leung, 2018; Turner et al., 2019) expressed the expectation that mathematical modelling would be an enabler of interdisciplinary practices and integrating professional disciplines with secondary school subjects and also in primary/elementary school (e.g., Baker and Galanti, 2015). However, from a literature survey of STEM integration practices 2013-2016, Bajuri et al. (2018) identified mathematical modelling as the least focused on integrative practice. They proposed

Preprint

using metacognition and social interaction development to promote these abilities in a mathematical modelling approach to STEM. Such a proposal would seem eminently sensible given the nature of mathematical modelling as conducted by professional modellers and in school classrooms and the fact modelling is included in each of the STEM disciplines and is “an opportunity to express and to develop disciplinary knowledge and ways of thinking” (Hjalmarson et al., 2020, p. 229).

5. Closing

As our preliminary findings from our systematic analysis of relevant literature show, all four threads we have explored have not been exhausted. These areas represent opportunities for the mathematics education research community to conduct further scholarly work and help advance the field at large. Our final report will also contribute to this.

References

J. B. Ärlebäck and L. Albarracín (2019). The use and potential of Fermi problems in the STEM disciplines to support the development of 21st century competencies. *ZDM Mathematics Education,* 51(6), 979–990.

M. R. Bajuri, S. M. Maat, and L. Halim (2018). Mathematical modeling from metacognitive perspective theory: A review on STEM integration practices. *Creative Education, 9*, 2203–2214. https://doi.org/10.4236/ce.2018.914161

W. Blum and D. Leiß (2007). How do students and teachers deal with modelling problems? In: C. Haines, P. Galbraith, W. Blum, and S. Khan (eds.), *Mathematical Modelling (ICTMA 12: Education, Engineering and Economics*. Chichester: Horwood, pp. 222–231.

W. Blum, P.L. Galbraith, H-W. Henn, and M. Niss (2007). *Modelling and Applications in Mathematics Education: The 14th ICMI Study*. New York: Springer.

L. Apostel (1972). *Interdisciplinarity: Problems of Teaching and Research in Universities.* Paris: Centre for Educational Research and Innovation of the Organization for Economic Cooperation and Development.

S. Carreira and A.M. Baioa (2015). Assessing the best staircase: Students’ modelling based on experimentation with real objects. In: K. Krainer and N. Vondrová (eds.) *Proceedings of CERME9.* Prague: ERME, pp. 834–840.

S. A. Chamberlin and B. Sriraman (eds). (2019). *Affect in Mathematical Modeling*. Cham: Springer.

Y. Chang, J. Krawitz, S., Schukajlow, and K-L. Yang (2020). Comparing German and Taiwanese secondary school students’ knowledge in solving mathematical modelling tasks requiring their assumptions. *ZDM Mathematics Education,* 52(1), 59–72.

J. C. Craig (2017). Real fantasies in mathematics education: Numeracy, quantitative reasoners, and transdisciplinary wicked problems. PhD thesis, Michigan State University, MI, USA.

J. A. Czocher (2018). How does validating activity contribute to the modelling process? *Educational Studies in Mathematics,* 99(2), 137–159.

J. Czocher, G. Stillman, and J. Brown (2018). Verification and validation: What do we mean? In: J. Hunter, P., Perger, and L. Darragh (eds.). *Proceedings of 41st annual*

preprint

conference of the Mathematics Education Research Group of Australasia (MERGA). Auckland: MERGA, pp. 250–257.

U. D’Ambrosio (1989). Historical and epistemological bases for modelling and implications for the curriculum. In: W. Blum, M. Niss and I. Huntley (eds.), *Modelling, Applications, and Applied Problem Solving*. Chichester: Ellis Horwood, pp. 22–27.

J. M. Diego-Mantecón, E. Haro, T. F. Blanco, and A. Romo-Vázquez (2021). The chimera of the competency-based approach to teaching mathematics: A study of carpentry purchases for home projects. *Educational Studies in Mathematics,* 107(2), 339–357.

J. M. Diego-Mantecón, T. Prodromou, Z. Lavicza, T. F. Blanco, and Z. Ortiz-Laso (2021). An attempt to evaluate STEAM project-based instruction from a school mathematics perspective. *ZDM Mathematics Education,* 53(5), 1137–1148.

B. Doig, J. Williams, D. Swanson, R. Borromeo Ferri, and P. Drake, eds. (2019). *Interdisciplinary Mathematics Education [ICME-13 monograph series]*. Springer Open.

R. Durandt, W. Blum, and A. Lindl (2022). Fostering mathematical modelling competency of first year South African engineering students: Which influence does the teaching design have? *Educational Studies in Mathematics,* 109(2), 361–381.

L. D. English (2007). Interdisciplinary learning in the primary mathematics curriculum. In: J. Watson and K. Beswick (eds.), *Proceedings of 30th annual conference of the Mathematics Education Research Group of Australasia (MERGA)*. Hobart: MERGA.

L. D. English (2008). Mathematical modelling: Linking mathematics, science and arts in the primary curriculum. In: B. Sriraman, C. Michelsen, A. Beckmann and V. Freiman (eds). *Proceedings of the Second International Symposium on Mathematics and its Connections to the Arts and Sciences* (MACAS2) (Centre for Science and Mathematics Education, Vol. 4). Odense, University of Southern Denmark, pp. 5−36.

L. D. English (2013). Complex modelling in the primary and middle school years: An interdisciplinary approach. In: G. A. Stillman, G. Kaiser, W. Blum and J. P. Brown (eds.), *Teaching Mathematical Modelling: Connecting to Research and practice.* Dordrecht: Springer, pp. 491–505.

L. D. English (2016). STEM education K-12: perspectives on integration*. International Journal of STEM Education*, 3:3. https://doi.org/10.1186/s40594-016-0036-1

L. D. English, J. B., Ärlebäck, and N. Mousoulides (2016). Reflections on progress in mathematical modelling research. In: Á. Gutiérrez., G.C. Leder, & P. Boero (eds.), *The Second Handbook of Research on the Psychology of Mathematics Education*. Rotterdam: Sense, pp. 383-413.

M. Frank, P., Richter, C., Roeckerath, and S. Schönbrodt (2018). Wie funktioniert eigentlich GPS? — ein computergestützter Modellierungsworkshop. In: G. Greefrath and H.-S. Siller (eds.), *Digitale Werkzeuge, Simulationen und mathematisches Modellieren.* Wiesbaden: Springer Spektrum, pp. 137–163.

P. Frejd and V. Geiger (2017). Exploring the notion of mathematical literacy in curricula documents. In: G. Stillman, G. Kaiser and W. Blum. (Eds.), *Mathematical Modelling and Applications: Crossing and Researching Boundaries in Mathematics Education.* Cham: Springer, pp. 255–264.

M. Gardner and J. W. Tillotson (2019). Interpreting integrated STEM: Sustaining pedagogical innovation within a public middle school context. *International Journal of Science and Mathematics Education,*17*(*7), 1283–1300.

A. M. Gibbs (2019). Socio-critical modeling and the role of Mathematics in society. PhD thesis, Florida Institute of Technology, FL, USA.

Preprint

H. H. L. Gonçalves and C. M. C. Pires (2014). Educação matemática na Educação Profissional de Nível Médio: análise sobre possibilidades de abordagens interdisciplinares. *Bolema (Boletim de Educação Matemática),* 28(48), 230–254.

R. Gresnigt, R. Taconis, H. van Keulen, K. Gravemeijer, and L. Baartman (2014). Promoting science and technology in primary education: A review of integrated curricula. *Studies in Science Education,* 50(1), 47–84.

C. Guerrero-Ortiz, H. R., Mejía-Velasco, and M. Camacho-Machínc (2016). Representations of a mathematical model as a means of analysing growth phenomena. *Journal of Mathematical Behavior*, 42, 109–126.

C. Hankeln (2020). Mathematical modeling in Germany and France: A comparison of students' modeling processes. *Educational Studies in Mathematics,* 103(2), 209–229.

M. A. Hjalmarson, N. Holincheck, C. K. Baker, and T. M. Galanti (2020). Learning models and modelling across the STEM disciplines. In: C.C. Johnson, M.J. Mohr-Schroeder, T.J. Moore, and L.D. English (Eds.), *Handbook of research on STEM education*. New York: Taylor & Francis, pp. 223–233.

C. R. Hirsch and A. Roth McDuffie, eds. (2016). *Mathematical Modeling and Modeling Mathematics.* USA: National Council of Teachers of Mathematics.

Y-S. Hsu and Y-F Yeh, eds. (2019). *Asia-Pacific STEM teaching practices: From theoretical frameworks to practices*. Singapore: Springer.

J. H. Jensen (2018). Mathematical modelling — hiding or guiding? In: C. Michelsen, A. Beckmann, V. Freiman, and U. T. Jankvist (eds.), *Mathematics as a Bridge between the Disciplines: Proceedings of MACAS — 2017 Symposium,* pp. 15–28.

C. C. Johnson, M. J. Mohr-Schroeder, T. J. Moore, and L. D. English, eds. (2020). *Handbook of research on STEM education*. New York: Taylor & Francis.

G. G. Joseph (2011). *The crest of the peacock: Non-European roots of mathematics* (3rd ed.). Princeton University Press.

T. R. Kelley and J. G. Knowles. (2016). A conceptual framework for integrated STEM education. *International Journal of STEM Education,* 3:11. https://doi.org/10.1186/s40594-016-0046-z

S. F. Kraus and E. Krause, eds. (2020). *Comparison of Mathematics and Physics Education I: Theoretical Foundations for Interdisciplinary Collaboration*. Wiesbadem: Springer Spektrum.

J. Krawitz, Y. -P. Chang, K-L. Yang, and S. Schukajlow (2022). The role of reading comprehension in mathematical modelling: Improving the construction of a real-world model and interest in Germany and Taiwan. *Educational Studies in Mathematics, 109*(2), 337–359.

F. K. S. Leung, G. A. Stillman, G. Kaiser, and K. L. Wong, eds. (2021). *Mathematical Modelling Education in East and West*. Cham: Springer.

Y. Li, K. Wang, Y. Xia, J. S. Froyd, and S. B. Nite (2020). Research and trends in STEM education: a systematic analysis of publicly funded projects. *International Journal of STEM Education,* 7:17. https://doi.org/10.1186/s40594-020-00213-8

K. Maass, K. V. Geiger, M. R. Ariza, and M. Goos (2019). The role of mathematics in interdisciplinary STEM education, *ZDM Mathematics Education,* 51(6), 869–884.

R. C. Miller (2020). Interdisciplinarity: Its meaning and consequence. In: *Oxford Research Encyclopedia, International Studies*. Oxford University Press. https://doi.org/10.1093/acrefore/9780190846626.013.92

T. J. Moore, A.C. Johnston, and A.W. Glancy (2020). STEM Integration: A Synthesis of Conceptual Frameworks and definitions. In: C.C. Johnson, M.J. Mohr-Schroeder, T. J. Moore, and L.D. English (Eds.), *Handbook of Research on STEM Education.* New York: Taylor & Francis, pp. 3–16.

preprint

National Academy of Sciences, National Academy of Engineering, and Institute of Medicine. (2005). *Facilitating Interdisciplinary Research*. Washington, DC: The National Academies Press.

M. Newman and D. Gough (2020). Systematic reviews in educational research: Methodology, perspectives and application. In: O. Ritcher- Zawacki., M. Kerres, S. Bedenlier, M. Bond., and K. Buntis. (Eds.), *Systematic Reviews in Educational Research: Methodology, Perspectives and Application* (pp. 41–54). Wiesbaden: Springer.

G. H. Roehrig, E. A. Dare, E. Ring-Whalen, and J. R. Wieselmann (2021). Understanding coherence and integration in integrated STEM curriculum. *International Journal of STEM Education, 8*:2 https://doi.org/10.1186/s40594-020-00259-8

G. Sala, V. Font, J. Giménez, and B. Barquero (2017). Inquiry and modelling in a real archaeological context. In: G.A. Stillman, W. Blum, and G. Kaiser (Eds.), *Mathematical modelling and applications: Crossing and researching boundaries in mathematics education* (pp. 325–335). Cham: Springer.

C. Sawatzki, A. Downton, and J. Cheeseman (2019). Stimulating proportional reasoning through questions of finance and fairness. *Mathematics Education Research Journal,* 31(4), 465–484.

G. A. Stillman, W. Blum, and M. S. Biembengut, eds. (2015). *Mathematical Modelling in Educational Research and Practice: Cultural, Social and Cognitive Influences*. Cham: Springer.

G. A. Stillman, W. Blum, and G. Kaiser, eds. (2017). *Mathematical Modelling and Applications: Crossing and Researching Boundaries in Mathematics Education*. Cham: Springer.

G. A. Stillman and J. P Brown, eds. (2019). *Lines of inquiry in mathematical modelling research in education [ICME-13 monograph series].* Springer Open.

G. A. Stillman, T. Ikeda, S. Schukajlow, J. L. Araújo, and J. B. Ärlebäck (submitted). Survey of Interdisciplinary Aspects of the Teaching and Learning of Mathematical Modelling in Mathematics Education. In: G. Greefrath, S. Carreira and G.A. Stillman (Eds.), *Advancing and Consolidating Research on Applications and Modelling in Mathematics Education*. Cham: Springer.

G. A. Stillman, G. Kaiser, W. Blum and J. P. Brown, eds. (2013). *Teaching Mathematical Modelling: Connecting to Research and Practice*. Dordrecht: Springer.

G. A. Stillman, G. Kaiser, and C. E. Lampen, eds. (2020). *Mathematical Modelling Education and Sense-making*. Cham: Springer.

J. Suh, M. Wickstrom, and L.D. English, eds. (2021). *Exploring the nature of early elementary modelling*. Cham: Springer.

O. Viirman and E. Nardi (2021). Running to keep up with the lecturer or gradual de-ritualization? Biology students' engagement with construction and data interpretation graphing routines in mathematical modelling tasks. *Journal of Mathematical Behavior,* 62, Article100858.

J. Williams and W. -M. Roth (2019). Theoretical perspectives on interdisciplinary mathematics education. In: B. Doig, J. Williams, D. Swanson, R. Borromeo Ferri, and P. Drake (eds.), *Interdisciplinary Mathematics Education [ICME-13 monograph series].* Springer Open, pp. 13–34.

Preprint